RESONANCE MEASUREMENT OF EVEN AND ODD MODE PROPAGATION CONSTANTS IN COUPLED MICROSTRIPS

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Abstract

The resonant properties of transmission-line resonators can be used to measure the line propagation constant [1,2]. This technique is applied here to the even and odd modes of propagation of symmetrical pairs of coupled microstrips. A sample measurement is presented and possible extensions in the use of the method are outlined.

1.- Introduction

Let us consider a low-loss transmission line section of length ℓ having characteristic impedance Z_c and propagation constant γ. Let the line be short-circuited at one end and fed through an admittance inverter of constant B_0, as illustrated in fig.1 .

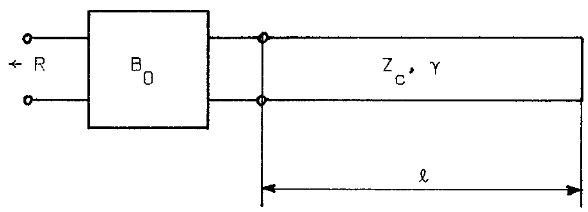

FIG. 1

The input reflection coefficient of the 1-port network shown in this figure with respect to an outside resistance R is given by

$$\rho = \frac{\mathrm{ctgh}\,\gamma\ell - n^2}{\mathrm{ctgh}\,\gamma\ell + n^2} \qquad (1)$$

where $n^2 = B_0 Z_c R$.

Let v_p be the phase velocity in the transmission line. It is easily shown that in the vicinity of every frequency satisfying:

$$\frac{\omega_s \ell}{v_p} = (2K+1)\frac{\pi}{2}\text{, K integer} \qquad (2)$$

the network approximately behaves as a series lumped resonant circuit. In fact, for $\omega \simeq \omega_s$ eq. (1) yields:

$$\rho \simeq \frac{r_0 + j\dfrac{\ell}{v_p}\dfrac{\omega - \omega_s}{\varepsilon + n^2}}{1 + j\dfrac{\ell}{v_p}\dfrac{\omega - \omega_s}{\varepsilon + n^2}} \qquad (3)$$

with $r_0 = (\varepsilon - n^2)/(\varepsilon + n^2)$ and $\varepsilon = \mathrm{tgh}\,\alpha\ell$. As usual, we denote by α the attenuation constant of the line.

On the other hand, the input reflection coefficient for a series lumped resonant circuit in the vicinity of its resonant frequency [3] is the same as (3) if we give the loaded Q the expression:

$$Q_L = \frac{\omega_s \ell}{2v_p}\,\frac{1}{\varepsilon + n^2} = \frac{(2K+1)\pi}{4(\varepsilon + n^2)}\,. \qquad (4)$$

For ω far from ω_s we have $|\rho| \simeq 1$ since the circuit is practically loss-free. However, for $\omega = \omega_s$ we have $\rho = r_0$, so that a resonant dip occurs if the constant B_0 is adjusted in such a way that $|r_0| << 1$. This allows a very accurate determination of ω_s.

The loaded Q can be easily found as well, since it is given by the expression $Q_L = \omega_s / \Delta\omega$, where $\Delta\omega$ is the frequency distance between the two 3-dB points.

Once Q_L and r_0 have been obtained, from (3) and (4) we get:

$$\varepsilon = (1 + r_0)\,(2K + 1)\,\frac{\pi}{8\,Q_L} \simeq \alpha\ell\,, \qquad (5)$$

since usually $\alpha\ell << 1$. Eq. (5) yields the attenuation constant, while eq. (2) gives:

$$v_p = \frac{2\,\omega_s \ell}{(2K + 1)\pi}\,. \qquad (6)$$

Thus the propagation constant γ can be completely determined by measuring ω_s, r_0 and $\Delta\omega$.

A similar procedure holds, of course, for an open-end transmission-line resonator, except that (2K+1) is replaced by 2K in all the preceding equations.

2.- Measurement procedure

Now we will show how to employ the foregoing measurement procedure in the special case of a symmetrical pair of coupled microstrips.

It is well known that a couple of symmetrical microstrips can support two different modes of propagation. The even mode has in-phase voltages on the two strips in every cross section, while for the odd mode the voltages are out of phase. If the strips are parallel connected at each end, only the even mode will propagate along the line, no matter how it is fed or loaded. Thus a transmission-line open-end resonator may be obtained, having the even-mode propagation constant, γ_E, and half the even-mode characteristic impedance, $Z_E/2$.

In order to practically realize the electrical situation of fig.1 , the admittance inverter is approximated by a small capacitive coupling. The latter is obtained by means of an air gap between the output tab of the R.F. connector and the microstrip. The position of the tab is adjusted by a dielectric screw to minimize $|r_o|$.

This is clearly shown by figs. 2 and 3. Before adjusting the position of the tab, the resonant dip is about 5 dB deep; a 25-dB dip is obtained after adjusting. The small change in resonant frequency (about 3%) is due to the imperfect realization of the admittance inverter, and can be theoretically explained.
Both figures refer to a couple of symmetrical microstrips whose geometry is defined by the following:

$W_1 = W_2 \simeq 0.9$ mm (common strip width)

$S \simeq 0.1$ mm (strip spacing)

$H \simeq 1.54$ mm (thickness of dielectric slab)

$\varepsilon_r \simeq 2.53$ (relative permittivity).

The strips were obtained by photo-etching a copper-plated rexolite substrate.

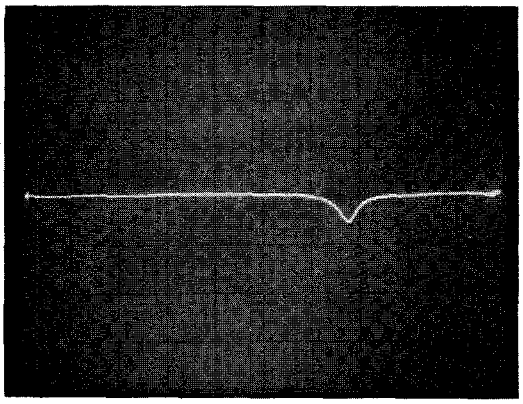

FIG. 2
Even-mode dip before adjusting inverter constant ($|r_o|$ is about -5 dB)

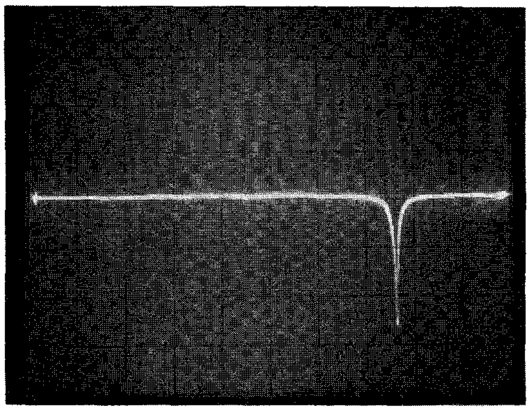

FIG. 3
Even-mode dip after adjusting inverter constant ($|r_o|$ is about -25 dB)

For both figures the scale factors are:

abscissa 50 MHz/div. (1 ÷ 1.5 GHz)
ordinate 10 dB/div.

The relevant parameters obtained from measurement are:

$$\omega_s = 2\pi \cdot 1.31 \text{ GHz}$$
$$\Delta\omega = 2\pi \cdot 16 \text{ MHz} ,$$

yielding:

$$v_{pE} = 2.05 \cdot 10^8 \text{ m/sec}$$
$$\alpha_E = 0.011 \text{ dB/cm.} \qquad (7)$$

The length of the resonator was 7.8 cm.

The odd mode of propagation is not directly accessible from the outside world. However, a situation similar to that of fig.1 can be obtained for this mode,too. First, let the strips be parallel connected at the input side, and separately loaded by the admittances Y_1, Y_2. This time the R.F. connector tab is soldered to the strips, so that no input admittance inverter exists.

Since the strips are fed in parallel, in the input section only the even mode is excited, while the odd mode is short-circuited. In the load section the even and odd modes are connected by a two-port network whose admittance matrix is:

$$\underline{Y}_{EO} = \frac{1}{2}\begin{bmatrix} Y_1+Y_2 & Y_1-Y_2 \\ Y_1-Y_2 & Y_1+Y_2 \end{bmatrix} . \qquad (8)$$

Now, if we choose:

$$Y_1 = -jB_0$$
$$Y_2 = jB_0 , \qquad (9)$$

where B_0 is a positive real number, then (8) becomes the admittance matrix of an admittance inverter of constant B_0. Thus when eqs. (9) hold the electrical situation in terms of even- and odd-mode transmission lines is as shown in fig.4 .

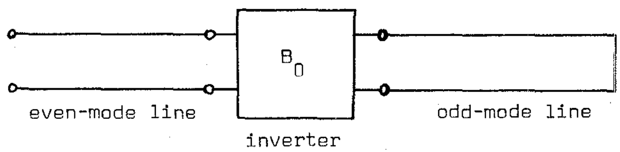

FIG. 4
Odd-mode resonant connection

Clearly, we are again faced with the same situation as in fig.1 , except that the short-circuited transmission-line resonator has now the odd-mode propagation constant γ_0. The even-mode transmission line merely acts as a cascade connection whose effects on the magnitude of the reflection coefficient are negligible, since it is very low-loss by eqs. (7).

The input reflection coefficient is measured in a $Z_E/2$-Ω system so that eqs. (5) and (6) may be directly used.

Now, in order that eqs. (9) hold, we just need to load line 2 by a short (<λ/4) transmission-line open-end stub of positive length, and line 1 by a similar stub of *negative* length. To obtain this in practice, we will shorten one of the strips, and lengthen the other, by a same amount Δℓ. The overall connection is schematically shown in fig.5 .

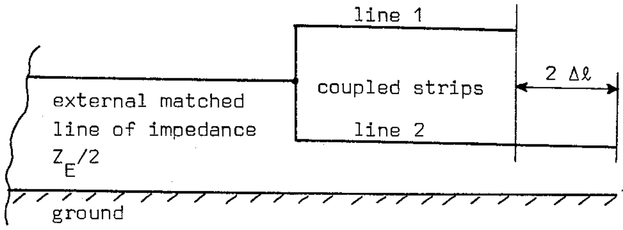

FIG. 5

Thus, making one of the strips slightly longer than the other is equivalent to connecting the even and odd modes in the load section by an admittance inverter. To minimize $|r_0|$ we only have to adjust the length Δℓ, since this is the physical parameter determining the inverter constant.

Fig.6 shows the first odd-mode resonant dip for the same two-strip line that was considered before. A 30-dB dip was obtained by making Δℓ ≃ 4 mm.

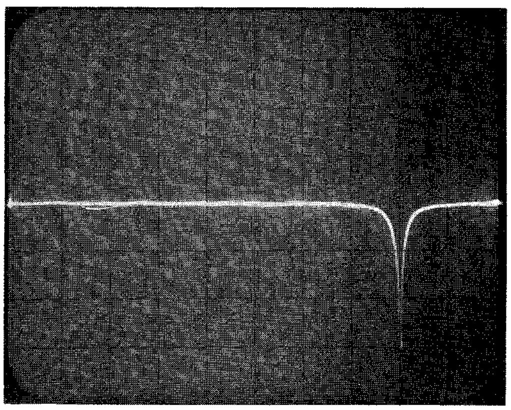

FIG. 6
Odd-mode dip for Δℓ ≃ 4 mm. ($|r_0|$ is about -30 dB)

The scale factors for this figure are:

abscissa 50 MHz/div. (0.5 ÷ 1 GHz)
ordinate 10 dB/div.

The relevant measured values are now:

$\omega_s = 2\pi \cdot 845$ MHz

$\Delta\omega = 2\pi \cdot 40$ MHz ,

from wich it is found:

$v_{p0} = 2.3 \cdot 10^8$ m/sec

$\alpha_0 = 0.024$ dB/cm.

For comparison with the even-mode values, we can make use of the standard $\sqrt{f}$ dependance, thus obtaining $\alpha_0 = 0.029$ dB/cm at 1.31 GHz.

3.- Conclusion

A measurement procedure for the even- and odd-mode propagation constants of symmetrical coupled microstrips has been outlined. The method relies upon the resonant properties of inverter-coupled transmission-line resonators. It is shown that this electrical configuration can be independently realized for both modes, thus allowing an easy determination of attenuation constant and phase velocity for each one.

The measurements are performed on a sample two-strip line typically a few centimeters long. The resonator is shielded so that no radiation losses [4] be included in the resulting attenuation constant.

For any given length of the sample line, the measurement can be performed at a number of discrete frequencies (i.e., the resonant frequencies of the particular resonator being used). To obtain data at other frequencies, samples of different lengths should be used. However, since the phase velocities and attenuation constants are usually well-behaved functions of frequency, interpolation of the discrete measured data gives very satisfactory results.

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